UDC 532

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Two models of vortex flows

Abstract. *Two models of vortex flows are proposed. One model is a vortex consisting of a laminar quasi-solid core and turbulent periphery. Compactness is ensured by the compensation of the vorticity field. As the vorticity field compensation condition true for both lamina and turbulent flow regimes, it may be used for this type of vortex. The second model is the turbulent vortex with a cavity when the pressure in the liquid drops to the saturated vapor pressure. The flow occurs in the annular region (annular turbulent vortex). Both models correspond to the currents observed in practice and seem plausible.*
Keywords: *compact vortex, laminar core of the vortex, turbulent periphery of the vortex, annular confined turbulent vortex*

Introduction

The concept of vortex compactness is interpreted differently. Many scientists identify the vortex with the vorticity field. If this field is compact, then the vortex is compact [1]. However, natural eddy currents actual measurements indicate that the real compact vortices can not have flow field where vorticity is of the same sign. Ultimate cases are the so-called quasi-point vortices - turbulent and laminar, - where the vorticity at axis should have a sign opposite to sign of the vorticity in the rest of the field. This allows us to meet the condition of the vorticity field compensation, which entails the compactness of the flow as a whole - and not just the vorticity field.

When considering quasi-point vortex models, it was pointed out [2], [3] that the velocity field on the axis of the vortex has a singularity. Many researchers consider instead of the velocity its circulation and avoid the singularity. In reality, the flow velocity can reach only finite values, determined by the pressure of saturated vapor of the liquid (at a given temperature). So there may be, at least, two flow scenarios. The first, when a vortex has even small in size but a finite laminar core and turbulent periphery and the pressure inside the vortex does not drop to the pressure of saturated vapors. The second scenario corresponds to the situation when a cavity forms inside the vortex. This means that the velocity in the vortex on its inner surface is so great that the pressure decreases to the saturated vapor pressure.

Compact vortex with laminar core and turbulent periphery

In many vortex flows, the inner part of the vortex, its core, is a stable flow, while the outer part, that is, the periphery, is unstable turbulent. Undoubtedly, there is an small transition region in the vortex, where the flow is no longer stable, but also not completely turbulent. However, in models this is not taken into account. The purpose of this model is a rough approximation of the real flow. In other words, the so-called zero approximation, which is specified as the initial condition in more complex models. ***But, nevertheless, this approximation takes into account the basic physical properties of the flow: the zero value of the azimuthal velocity on the axis of the vortex and on its outer boundary (compactness), the laminar core and the turbulent periphery.***

If we consider the corresponding Gromeka-Lamb equations (often confused with the Navier-Stokes equations) in cylindrical coordinates, then, taking into account the laminar flow in the core and turbulent periphery, we obtain the following model for azimuthal velocity and vorticity (axial component) :

$$V_\theta = \begin{cases} \dfrac{\Gamma}{2\pi\varepsilon^2} r, \quad 0 \le r \le \varepsilon; \\ \dfrac{\Gamma}{2\pi(R_V - \varepsilon)}\left(\dfrac{R_V}{r} - 1\right), \quad \varepsilon \le r \le R_V; \\ 0, \ r \ge R_V. \end{cases} \quad \omega_z = \begin{cases} \dfrac{\Gamma}{\pi\varepsilon^2}, \quad 0 \le r < \varepsilon; \\ -\dfrac{\Gamma}{\pi(R_V - \varepsilon)}\dfrac{1}{r} \quad, \quad \varepsilon \le r < R_V; \\ 0, \ r \ge R_V. \end{cases} \tag{1}$$

These relations are valid when the pressure in the core of the vortex does not drop to saturated vapor pressure. If the azimuthal velocity in the vortex is so high that an air cavity is formed, the core of the vortex is already absent and only the turbulent periphery remains - the annular vortex.

Turbulent confined annular vortex with a cavity at the axis of rotation

To construct a vortex model with a cavity, we use the solution of the corresponding equation describing the turbulent rotation [3,4]

From the foregoing, a simple model of a turbulent annular vortex follows

$$V_\theta = \begin{cases} cavity, \quad 0 \le r \le \varepsilon; \\ \dfrac{C_1}{r} + C_2, \quad \varepsilon \le r \le R_V. \end{cases} \qquad \omega_z = \begin{cases} cavity, \quad 0 \le r \le \varepsilon; \\ \dfrac{C_2}{r}, \quad \varepsilon \le r < R_V. \end{cases} \tag{2}$$

Unlike the first model, the motion occurs in a finite (confined) area (pipe-like). Integrally, the circulation in the vortex is not equal to zero. Vorticity field is no longer compensated. The main task for this flow is the determination of the radius of the cavity. This is achieved by the condition that the pressure in the vortex drops to the value of the saturated vapor pressure, i.e.

$$V_\theta(r = \varepsilon) = \frac{C_1}{\varepsilon} + C_2 = \sqrt{\frac{2(P_{at} - P_{sv})}{\rho}} \ . \tag{3}$$

To determine both constants, in addition to relation (3), it is necessary to meet the non-slip condition on the outer wall

$$V_\theta(r = R_V) = 0 \quad \Leftrightarrow \quad \frac{C_1}{R_V} + C_2 = 0, \quad \Rightarrow \quad C_1 = -R_V C_2. \tag{4}$$

Relations (3-4) allow us to find the constants C_1 and C_2. In explicit form, we obtain the following simple model of a constrained vortex with a cavity at the axis of rotation region

$$V_\theta = \begin{cases} cavity, \quad 0 \le r \le \varepsilon; \\ \left(\dfrac{1 - \dfrac{R_V}{r}}{1 - \dfrac{R_V}{\varepsilon}}\right)\sqrt{\dfrac{2(P_{at} - P_{sv})}{\rho}}, \quad \varepsilon \le r \le R_V. \end{cases} \qquad \omega_z = \begin{cases} cavity, \quad 0 \le r \le \varepsilon; \\ \dfrac{1}{1 - \dfrac{R_V}{\varepsilon}}\sqrt{\dfrac{2(P_{at} - P_{sv})}{\rho}}\dfrac{1}{r}, \quad \varepsilon \le r \le R_V. \end{cases} \tag{5}$$

In this case, the radius of the cavity ε is determined from the relation

$$V_\theta(r = \varepsilon) = \frac{\Gamma}{2\pi\varepsilon} = \sqrt{\frac{2(P_{at} - P_{sv})}{\rho}} \quad \Rightarrow \varepsilon = \frac{\Gamma}{2\pi}\sqrt{\frac{\rho}{2(P_{at} - P_{sv})}}.$$

Figures 1—2 show the fields of velocity and vorticity corresponding to both of the above models.

Thus, from a simple model of a compact turbulent vortex, one can obtain flow models corresponding to realities. In conclusion, it is important to note that a turbulence model with a constant viscosity coefficient was used, which is valid for flows with Reynolds numbers of the order of a million — the so-called Ultimate Taylor – Couetter flow [4].

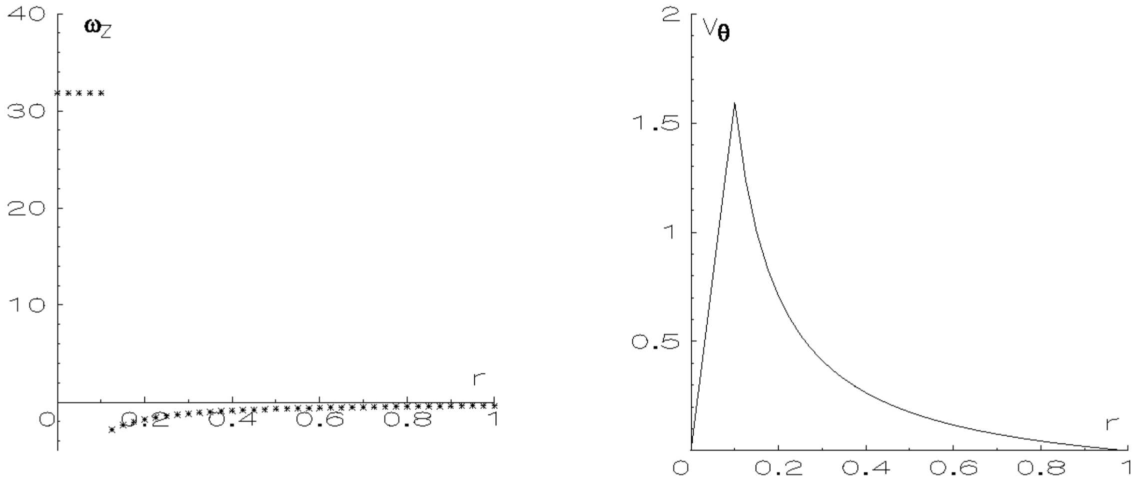

Fig. 1. The field of vorticity and azimuthal velocity in a compact vortex with a laminar core and turbulent periphery.

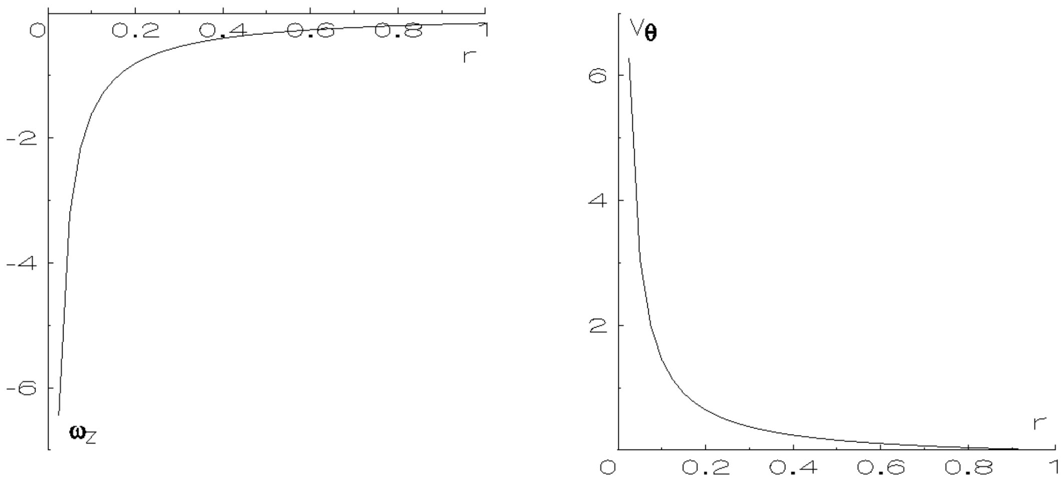

Fig. 2. Fields of vorticity and azimuthal velocity in a confined annular vortex with a cavity of rotation.

References

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